CHAPTER 2

Newton's Gravity

One Force. Two Phenomena. One Equation.

At the end of Chapter 1, we left humanity in an interesting position. By 1642, scientists knew a great deal about gravity. Galileo had shown that all objects fall with the same acceleration. Kepler had shown that planets sweep out precise mathematical patterns in their orbits. But nobody had connected these two facts. Nobody had asked: could the force pulling an apple downward and the force keeping the Moon in orbit be the same thing?

That question was answered by a 23-year-old student who was sent home from Cambridge University in 1665 because of a plague outbreak, and who spent the next 18 months alone on his family farm in Lincolnshire doing some of the most consequential thinking in the history of science.

His name was Isaac Newton. And what he realized, staring at the sky one evening -- perhaps with an apple tree nearby, perhaps not -- was something genuinely astonishing.

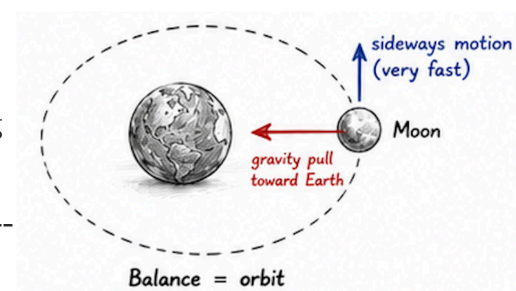
The Apple and the Moon

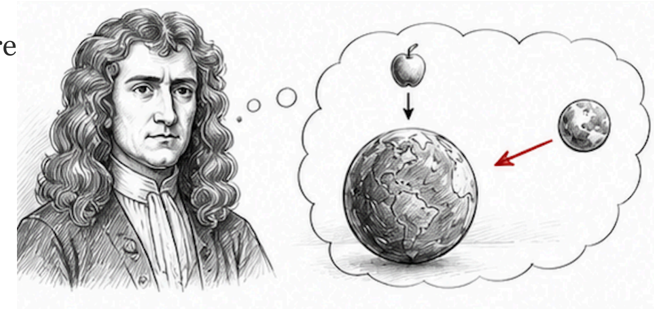
Here is Newton's key insight, stated simply: the Moon is falling.

Not falling toward the Earth in the way a dropped stone falls -- the Moon is not getting closer. But it is constantly being pulled toward Earth, constantly accelerating toward Earth's center. The reason it doesn't crash into us is that it is also moving sideways -- very fast. By the time gravity has pulled it a little bit downward, it has also moved so far sideways that the Earth's curved surface has dropped away beneath it by exactly the same amount. The Moon perpetually falls toward Earth and perpetually misses it. That is what an orbit is.

> **Think About It**
> *The Moon is about 384,000 km from Earth. Nothing is holding it up. Nothing is pushing it sideways. Left alone in empty space, it would fall straight toward Earth. So why has it been orbiting for four and a half billion years without getting any closer? What keeps it moving sideways?*

The answer -- which we will build up to carefully -- is that the Moon's sideways speed was given to it when the solar system formed, and in the vacuum of space, there is nothing to slow it down. Gravity curves its path into a circle, but gravity alone cannot stop the sideways motion. The Moon is trapped in a perpetual balance between falling and moving -- and that balance is an orbit.





But Newton's deeper realization was this: if the Moon is being pulled toward Earth, and if apples are being pulled toward Earth, then perhaps the same force is responsible for both. Not two different phenomena requiring two different explanations -- one force, reaching across 384,000 km of empty space, doing both jobs simultaneously.

To test this idea, Newton needed to know how gravity weakens with distance. And that question leads us to the concept of a gravitational field.

> **Watch & Explore**
>
> If you have not watched the apple and the moon episode, now is the time to do it!
>
> \> [The Mechanical Universe -- Episode 8: The Apple and the Moon]

The Gravitational Field

Before we write a single equation, we need to build a mental picture -- because the picture is more important than the math. The concept we need is the gravitational field.

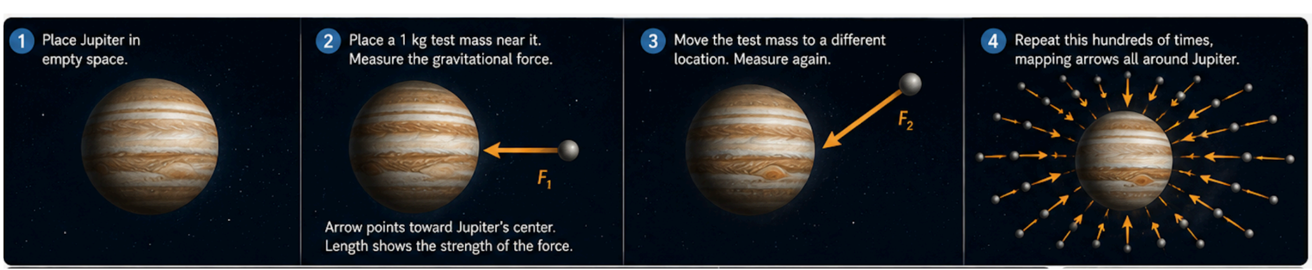
Every object with mass produces an invisible influence in the space around it. This influence extends in all directions, weakening with distance, and pulls any other mass it encounters toward the object that created it. This invisible region of influence is the gravitational field.

The field concept was not actually Newton's idea -- it was developed more formally by Michael Faraday in the 19th century. But it is the best mental tool we have for understanding gravity, so we will use it from the start.

Mapping the Field: A Thought Experiment

Here is how to make the gravitational field visible in your mind. Imagine placing Jupiter -- a massive planet -- in otherwise empty space. Now, place a small 1 kg test mass near it. Measure the gravitational force Jupiter exerts on it: note the strength and direction, and draw an arrow. The arrow points toward Jupiter's center. Its length represents the strength of the force.

Now move the test mass to a different location and measure again. Draw another arrow. Repeat this hundreds of times, building up a map of arrows all around Jupiter.



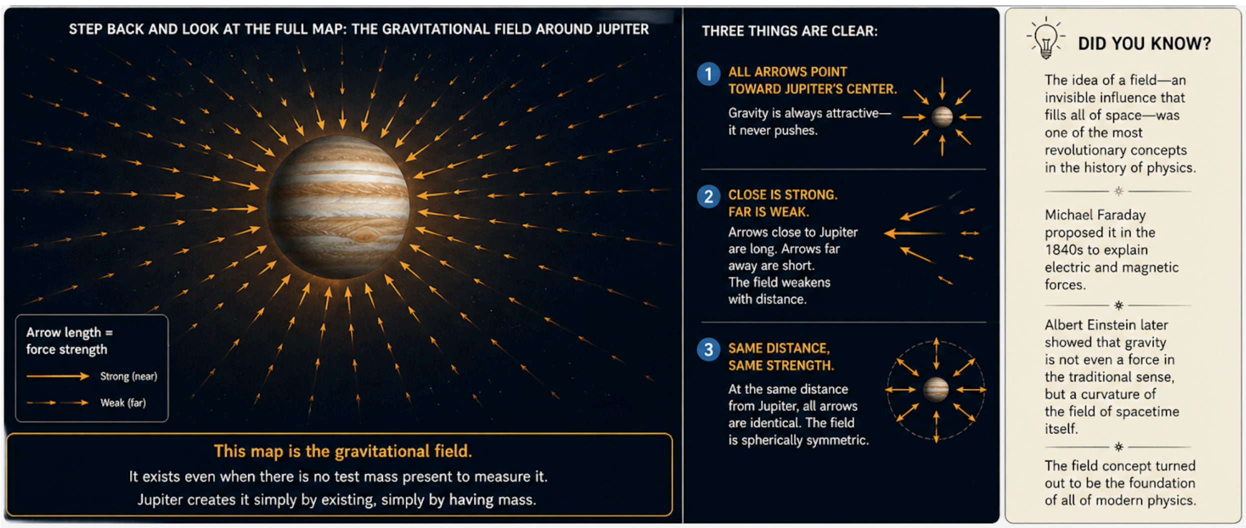
When you step back and look at the full map, three things are immediately clear:

* Every arrow points toward Jupiter's center. Gravity is always attractive -- it never pushes.
* Arrows close to Jupiter are long. Arrows far from Jupiter are short. The field weakens with distance.
* At the same distance from Jupiter, all arrows are identical -- the field is spherically symmetric.

This map is the gravitational field. It exists even when there is no test mass present to measure it. Jupiter creates it simply by existing, simply by having mass.

Did You Know?

The idea of a field -- an invisible influence that fills all of space -- was one of the most revolutionary concepts in the history of physics. Michael Faraday proposed it in the 1840s to explain electric and magnetic forces. Albert Einstein later showed that gravity is not even a force in the traditional sense, but a curvature of the field of spacetime itself. The field concept turned out to be the foundation of all of modern physics.

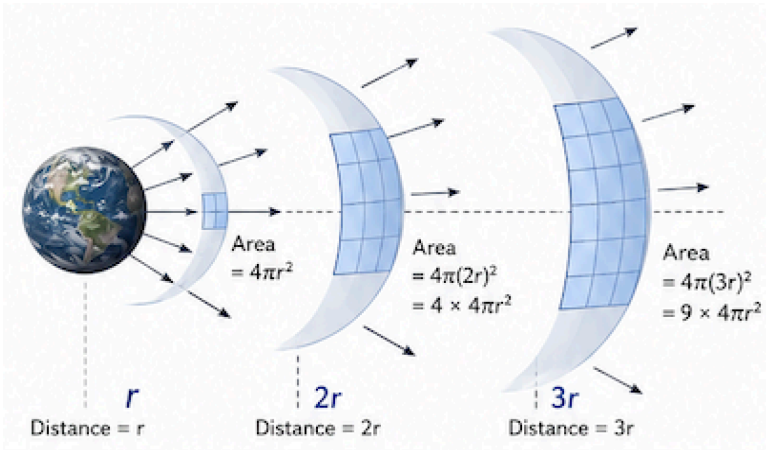


How the Field Depends on Distance

Now for the most important property of the gravitational field: how does it weaken as you move away from the massive object?

The answer is the inverse-square law. Here is the intuitive way to understand it. Imagine Jupiter's gravitational field spreading outward in all directions like light from a lamp. This field has to spread over a larger and larger surface as you move away. A sphere's surface area grows as r^2, where r is the radius. So the field available per unit area drops as 1 divided by r^2.

This is why all spreading-through-space phenomena follow an inverse-square law. Light, gravity, electric force, sound from a point source, all obey the same geometry of three-dimensional space.



Here is what the inverse-square law means in practice:

Change in distance	Distance multiplied by	Force divided by	Example
Double the distance	x 2	$4 = (2^2)$	Move from 1r to 2r: force drops to 1/4
Triple the distance	x 3	$9 = (3^2)$	Move from 1r to 3r: force drops to 1/9
Ten times farther	x 10	$100 = (10^2)$	Force drops to 1/100
Half the distance	x 1/2	$¼ = (½)^2$	Force quadruples
One tenth the distance	x 1/10	$1/100 = (1/10)^2$	Force multiplies by 100

Think About It

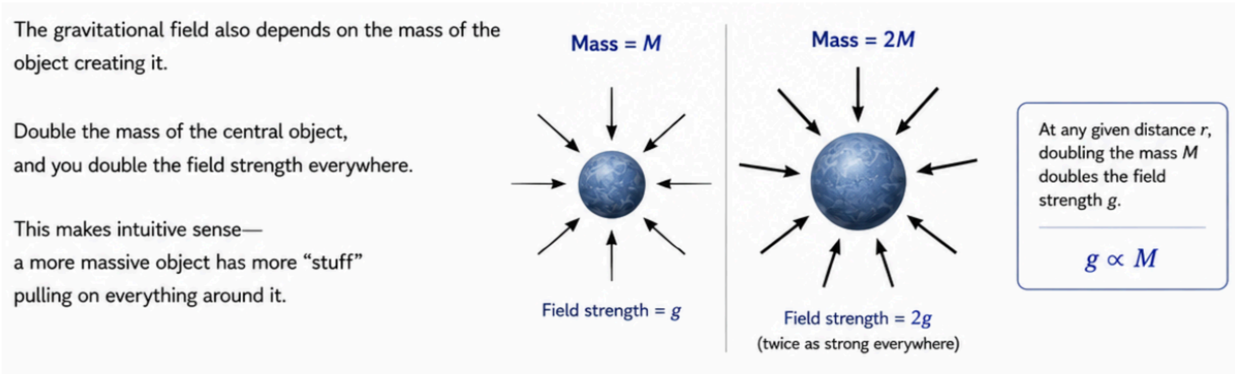
Gravity follows a $1/r^2$ law -- it never actually reaches zero, no matter how far you travel. Does this mean the Earth's gravity reaches the Moon? The Sun? The Andromeda Galaxy, 2.5 million light years away? Pause and think about what this implies about the reach of every massive object in the universe.

The answer is yes -- in principle, gravity reaches everywhere. The Earth's gravitational field extends to the Moon, to the Sun, to Andromeda. It just becomes unimaginably small at great distances. Gravity has an infinite range. It simply never fully switches off.

How the Field Depends on Mass

The gravitational field also depends on the mass of the object creating it. Double the mass of the central object, and you double the field strength everywhere. This makes intuitive sense -- a more massive object has more 'stuff' pulling on everything around it.

Combining both observations, we recognize that the gravitational field strength g at any point in space is proportional to the distance r^2 from a mass M and the Mass itself:



STEP 1 -- The Pattern We Observed

g proportional to M / r^2

g increases with mass M | g decreases with the square of distance r | This is the pattern -- not yet a precise equation

This proportionality is a genuine discovery. It tells us the shape of the relationship -- how g changes when M changes, and how g changes when r changes. It is a pattern, and patterns are exactly what physicists look for. But a proportionality, however elegant, cannot tell you the actual value of g at a specific location. It can only tell you that if you double M, g doubles -- or if you double r, g drops to one quarter. It cannot tell you: how strong gravity is pulling on this object, right here, right now.

To answer that question, you need to turn the proportionality into a true equation. And to do that, you need one more ingredient: a constant of proportionality.

Think About It

Here is a proportionality you might recognize. The longer a spring is stretched, the stronger it pulls back. Force is proportional to extension: F proportional to x. But this proportionality alone cannot tell you how many Newtons of force a specific spring exerts when stretched by 5 cm. What additional piece of information would you need to turn this into a true equation? Where would that information come from -- theory, or measurement?

In the case of gravity, the constant of proportionality is G -- the Universal Gravitational Constant. Notice the word universal. G is not a property of Earth, or of any particular planet, or of any particular material. It is a fixed number that sets the fundamental strength of gravity throughout the entire universe -- on Earth, on the far side of the Milky Way, in galaxies billions of light years away. Wherever there is mass, G tells you how strongly gravity acts.

But here is the crucial point: G cannot be derived from theory. It cannot be calculated from first principles. It has to be measured. Someone -- somewhere, sometime -- had to go into a laboratory and measure the actual gravitational force between objects of known mass at a known distance, and extract the number that makes the equation balance. That is the only way to find G. It is a fact about our universe that has to be read from nature, not reasoned from mathematics.

With G measured and in hand, the proportionality becomes a true equation:

STEP 2 -- The Equation: Gravitational Field Strength

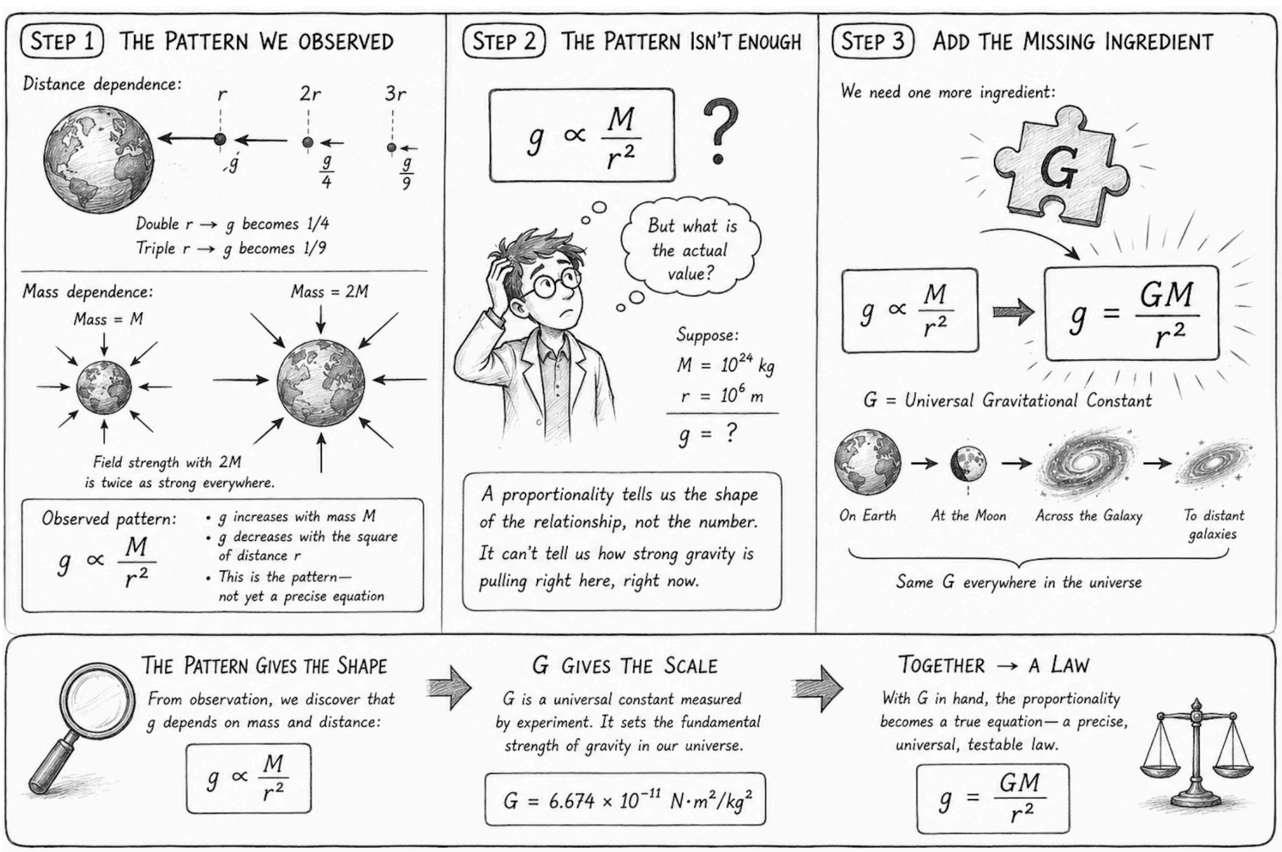
g = G * M / r^2

*g = field strength (m/s^2) | G = 6.674 x 10^-11 N*m^2/kg^2 | M = mass of source object (kg) | r = center-to-center distance (m)*

Now we can calculate. Now we can put in numbers and get numbers out. The proportionality told us the shape of the relationship. G tells us the scale. Together they give us a law -- a precise, universal, testable prediction about the strength of gravity at any point in space near any mass in the universe.

This two-step process -- proportionality first, then constant of proportionality -- is not unique to gravity. It is how most of the great laws of physics were discovered. Coulomb's Law for electric force, the law of radioactive decay, Hooke's Law for springs, Newton's Second Law itself -- all of them started as observed proportionalities, and all of them required a measured constant to become precise equations. The pattern comes from observation. The constant comes from careful experiment.

Neither alone is enough.



Did You Know?

The value of G is known far less precisely than almost any other fundamental constant in physics. The speed of light is known to 9 significant figures. The charge of the electron is known to 10 significant figures. But G is only reliably known to about 5 significant figures -- and different laboratory measurements still disagree slightly with each other. The reason is exactly what Cavendish discovered in 1798: the gravitational force between human-scale objects is so unimaginably small that measuring it with high precision remains one of the hardest experimental challenges in all of physics. The story of how Cavendish first measured G -- and why it was such a monumental achievement -- is the subject of the next section.

Why don't you move towards your water bottle?

Every object with mass — you, your book, your cup, this page — creates its own gravitational field and attracts everything around it. So why don't you feel pulled toward the cup on your desk?

🤔 Think About It

Think carefully before reading the answer. Most people — including some physics teachers — get this wrong.

⚠️ Common Misconception

✗ **People often say:** *"The Earth's gravity is so strong it overwhelms the cup's gravity, so we can't feel the cup's pull."*

✓ **The truth is:** Earth's gravity pulls you downward. The cup's gravity pulls you sideways toward the cup. These are two completely separate forces in different directions — Earth's gravity does not cancel the cup's pull. The real reasons are: (1) gravity is extraordinarily weak — Big G is only 6.674×10^{-11} — making the cup's pull on you negligibly small; and (2) friction between your feet (or your seat) and the floor prevents any tiny sideways pull from moving you.

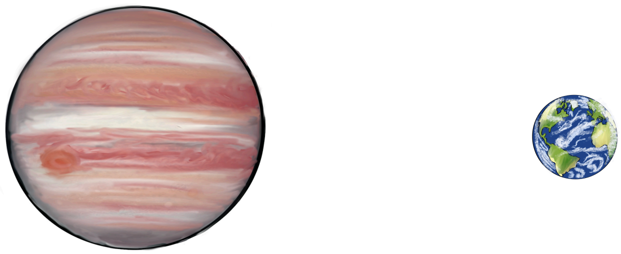
Here is a thought experiment to make this crystal clear. Imagine being transported with just your cup into deep space, far from any planet. Now there is no Earth gravity, no friction. Would you feel the cup's pull? Would you move toward it?

Yes! In the absence of friction, you and your cup would very gradually drift toward each other. If your cup were just 1 meter away, the two of you would collide — in a matter of hours. Gravity is always there; it is friction that makes it impossible for the two objects to float towards each other.

Both masses appear in Newton's gravitational force- what does it mean?

Notice that both masses appear in the numerator -- m1 and m2 multiplied together. This encodes a beautiful symmetry: the gravitational force between two objects depends equally on both of them.

Jupiter pulls on Earth with the same force that Earth pulls on Jupiter. The apple pulls on the Earth just as hard as the Earth pulls on the apple -- just in the opposite direction.



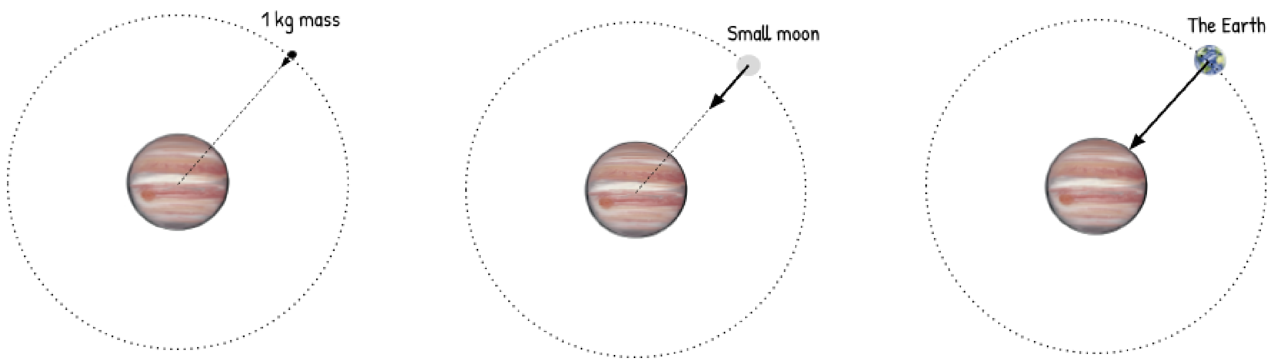
In this picture, Jupiter and Earth are exerting a gravitational pull on each other. Which planet applies a larger force on the other? Jupiter has a larger mass; does it exert a greater force on the Earth?

Based on Newton's third law, we know that when one object applies a force to the second object, the second object applies an equal and opposite force on the first. This means that the gravitational attraction between Jupiter and Earth must be exactly equal and opposite in direction. But let's dig a little deeper to understand exactly why these two forces are equal and opposite to each other.

Earlier, we learned that any object with mass creates a gravitational field that influences all other masses. Now, let's explore what happens when another mass enters the gravitational field of the first one.

If you bring a mass, like Earth, near Jupiter, the gravitational force between them increases directly proportional to the mass of the Earth. This may be surprising, but this is what happens!

For example, a bowling ball in Jupiter's gravitational field will experience a force that is directly proportional to its mass; the mass of a bowling ball. Similarly, a small moon and Earth feel forces directly proportional to their masses.



Let's calculate the forces on each object in the gravitational field of Jupiter.

Remember, gravitational field strength (g) depends on the planet creating the field—in this case, Jupiter. So when different objects enter Jupiter's field—for example, the 1 kg mass—the force of gravity depends on $m_{1\,kg}$ and Jupiter's gravitational field strength ($g_{Jupiter}$). Therefore,

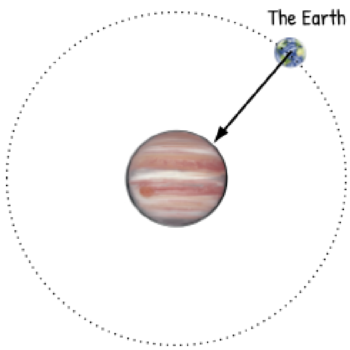
$F_{1kg} = m_{1\,kg} \cdot g_{Jupiter}$ $F_{small\ moon} = m_{small\ moon} \cdot g_{Jupiter}$ $F_{Earth} = m_{Earth} \cdot g_{Jupiter}$

But $g_{Jupiter}$ also depends on the mass of Jupiter. So, it is clear that the F_g is proportional to the masses of both objects. This creates a beautiful symmetry that is consistent with Newton's third law. Let's explore this sentence a little deeper.

First, we'll calculate the force due to Jupiter's gravity on Earth. Then we'll flip it and see what force Jupiter feels due to Earth's gravity.

To calculate this force, we must first determine Jupiter's gravitational field strength at the Earth's center. To do so, we draw a circle centered on Jupiter that passes through the center of the Earth. This will give us the center-to-center distance of the two objects.

Next, we have to multiply the Earth's mass (m_{Earth}) by Jupiter's field strength at that location ($g_{Jupiter}$), which is the same as $G.m_{jupiter}/distance^2$.

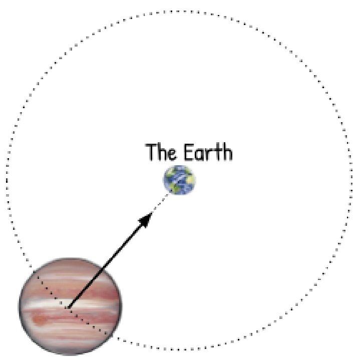


$F_g = m_{Earth} \cdot g_{Jupiter}$

$F_g = m_{Earth}(G \,.\, m_{jupiter}/distance^2)$

$$\boxed{F_g = G \,.\, m_{jupiter} \,.\, m_{Earth}/distance^2}$$

Next, let's calculate Earth's gravitational pull on Jupiter. To do so, we have to draw a circle centered on Earth that passes through Jupiter's center. Then, we have to multiply Jupiter's mass ($m_{Jupiter}$) by Earth's gravitational field strength at the center of Jupiter (g_{Earth}), which is the same as $G \,.\, m_{Earthr}/r^2$.



$F_g = m_{Jupiter} \cdot g_{Earth}$

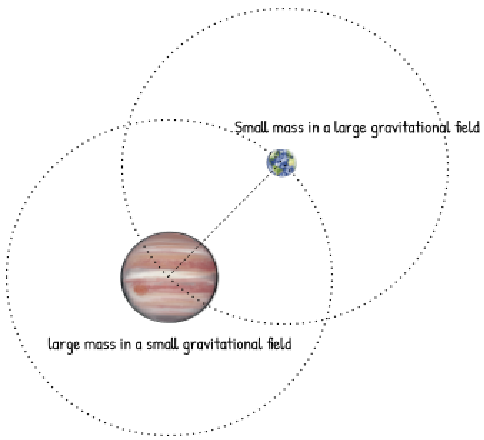
$F_g = m_{Jupiter}(G \,.\, m_{Earth}/distance^2)$

$$\boxed{F_g = G \,.\, m_{jupiter} \,.\, m_{Earth}/distance^2}$$

Do you see how the force of gravity is the same in both cases?

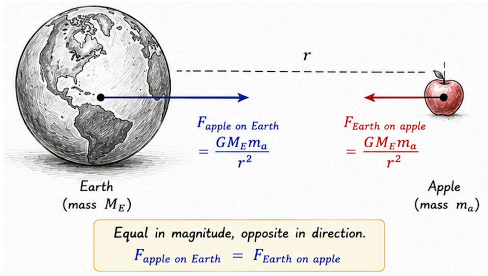
And this, ladies and gentlemen, is the reason why the pull of Jupiter on Earth is exactly equal and opposite to the pull of Earth on Jupiter, consistent with Newton's third law.

When two objects interact with each other gravitationally, the smaller mass in the larger mass's gravitational field produces the same force as the larger mass in the smaller mass's gravitational field.



$m_{jupiter}m_{Earth} = m_{Earth}m_{jupiter}$ It is like saying 2x5 equals 5x2. Some constants are multiplied on both sides, but those constants are the same on both sides of the equation. It is like saying 3x2x5 equals 3x5x2. The distance between the two planets is the same, and the "Big G" is the same, making the gravitational force of attraction the same.

A quick check of your understanding: There is an apple near the surface of the Earth. The Earth is pulling the apple towards its core. Does the apple pull the Earth more, less, or with an equal force towards its center? Pause, think, and answer. The apple pulls the Earth as much as the Earth pulls the apple! The gravitational force on a small mass in the larger mass's gravitational field is the same as the force on a larger mass in the smaller mass's gravitational field.



This is not obvious. You might think that because the Earth is so much more massive, it does most of the work. But Newton's Third Law of motion guarantees the symmetry: for every force there is an equal and opposite force. The apple pulls Earth upward with 1 Newton, and Earth pulls the apple downward with 1 Newton. The reason Earth does not visibly accelerate toward the apple is that its mass is enormous -- F = ma means the same force produces a vanishingly tiny acceleration in such a massive object.

Think About It

The Sun pulls on Earth with a gravitational force of about 3.6 x 10^22 Newtons. Earth pulls on the Sun with the same force. The Sun has about 333,000 times Earth's mass. So why does the Earth orbit the Sun, and not the other way around? Think carefully -- both objects are being pulled with the same force.

Both objects actually orbit their common center of mass. Because the Sun is so much more massive, the center of mass lies deep inside the Sun -- close to its center -- so the Sun wobbles only very slightly while Earth traces a wide circle. But the wobble is real, and astronomers use exactly this technique to detect planets orbiting distant stars: they look for tiny wobbles in the star's motion caused by the gravitational tug of an unseen planet.

Did You Know?

Gravity is the weakest of the four fundamental forces of nature -- weaker than electromagnetism by a factor of about 10^36 (a 1 followed by 36 zeros). The reason gravity dominates our everyday experience is that it has infinite range and is always attractive, while electric and magnetic forces can cancel each other out. On the scale of planets, stars, and galaxies, gravity wins by default because everything else has already canceled.

Gravity on a Graph: The Inverse-Square Law in Action

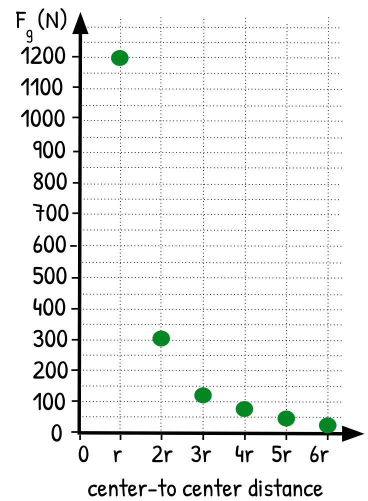
Words and ratios can only take you so far. Let us put numbers into the inverse-square law and see what it actually looks like. We will use a starting force of 1200 N at distance r -- a round number chosen purely to make the arithmetic clean.

Part 1 -- Moving Away: Force Drops Fast

What happens to the gravitational force as we move farther away -- to 2r, 3r, 4r, and beyond? Each time the distance multiplies, the force divides by the square of that multiplier.

Distance	r squared	Gravitational Force Fg	What happened?
r	1	1200 N	Starting point
2r	4	1200 / 4 = 300 N	4x weaker
3r	9	1200 / 9 = 133 N	9x weaker
4r	16	1200 / 16 = 75 N	16x weaker
5r	25	1200 / 25 = 48 N	25x weaker
10r	100	1200 / 100 = 12 N	100x weaker
100r	10,000	1200 / 10,000 = 0.12 N	10,000x weaker

Notice how quickly the force collapses. Doubling the distance cuts the force to one quarter. Going ten times farther cuts it to one hundredth. Going a hundred times farther cuts it to one ten-thousandth. This is not a gradual fade -- it is a steep cliff.



> **Think About It**
> *Look at the table. Between 1r and 2r, the force drops by 900 N. Between 4r and 5r, it drops by only 27 N. The force is changing fastest when you are closest. Why does the inverse-square law produce this pattern -- large changes close in, tiny changes far out?*

The graph of this data is the signature shape of the inverse-square law -- a curve that plunges steeply near the origin and then flattens out as distance grows. You will see this same curve again in electromagnetism, in the intensity of light, and in the strength of radio signals. It is one of the most important shapes in all of physics.

Part 2 -- Moving Closer: Force Rockets Up

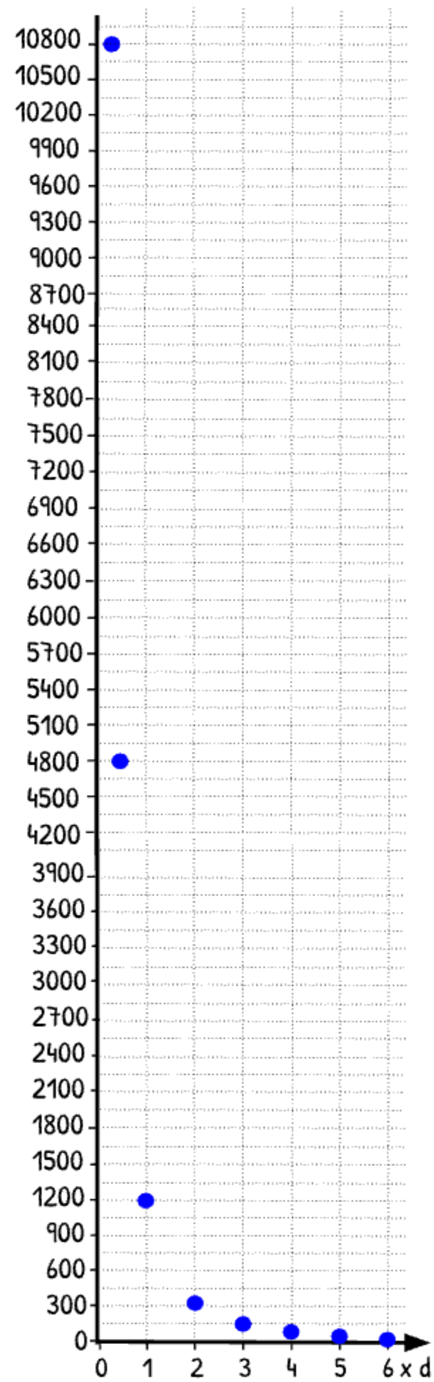
Now let us go the other way -- cutting the distance in half, then in thirds, then in quarters. Halving the distance replaces r^2 in the denominator with (r/2)^2 = r^2/4. Dividing by a quarter is the same as multiplying by 4. The force quadruples.

> **Algebra Tip**
> Dividing by a fraction = multiplying by its reciprocal.
>
> 1200 / (1/4) = 1200 x 4 = 4,800 N
> 1200 / (1/9) = 1200 x 9 = 10,800 N
> 1200 / (1/16) = 1200 x 16 = 19,200 N
>
> Ask yourself: how many quarters fit into 1200? The answer is 4,800.

Here is the combined table, showing the full picture -- what happens as you move both closer and farther away from the starting distance r.

Distance	r squared	Fg (starting = 1200 N)	What happened?
r/4	r^2 / 16 = 1/16	1200 / (1/16) = 19,200 N	16x stronger
r/3	r^2 / 9 = 1/9	1200 / (1/9) = 10,800 N	9x stronger
r/2	r^2 / 4 = 1/4	1200 / (1/4) = 4,800 N	4x stronger
r	**1**	**1200 N**	**Starting point**
2r	4	300 N	4x weaker
3r	9	133 N	9x weaker

Look at the top of the table. At r/4, the force is 19,200 N -- sixteen times the starting value of 1,200 N. That point cannot even fit on the same graph as the data for 2r and 3r. I will have to go to the top of the next page to fit that point on the same graph. The curve becomes nearly vertical as you approach the source. This is why gravity near a very dense object -- like a neutron star or a black hole -- becomes so extreme so quickly.



Did You Know?

The force between two electrons follows exactly the same inverse-square law as gravity -- but it is electric force, not gravitational force. When you push your hand against a table, the table pushes back not because of gravity but because the electrons in the table's atoms repel the electrons in your hand's atoms, and that electric repulsion follows 1/r^2. You cannot walk through walls because of the same mathematical law that keeps the Moon in orbit.

Think About It

The graph for decreasing distances shoots almost straight up near the origin. But can the distance r ever actually reach zero for two solid objects? What is the minimum possible distance between, say, two lead balls? What does this tell us about whether the force can ever become truly infinite?

Two solid objects can only get as close as the sum of their radii -- they cannot overlap. So r never reaches zero for real objects, and the force never becomes infinite. The curve shoots upward steeply, but it always stops at some minimum distance set by the physical size of the objects involved. For a black hole, where all the mass is compressed to a mathematical point, the story is very different -- and we will return to that in Chapter 3.

What the Graph Is Telling You

The inverse-square law graph has three key features to remember:

* **It falls steeply near the origin.** Small increases in distance produce enormous drops in force when you are already close.
* **It flattens at large distances.** Large increases in distance produce barely noticeable changes in force when you are already far away.
* **It never reaches zero.** No matter how far you travel, gravity always remains -- infinitesimally small, but never completely absent.

Thinking About the Graph

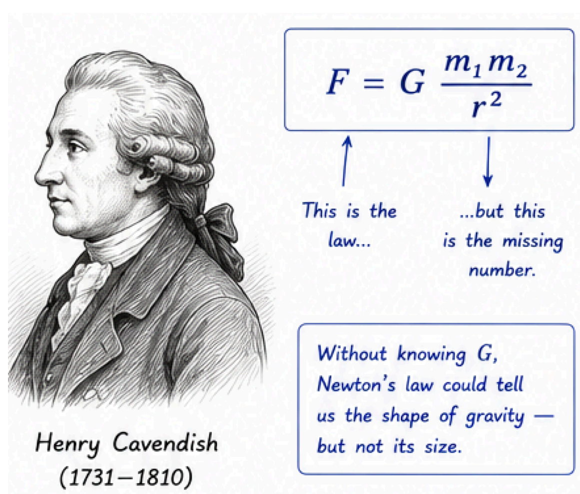
* The graph of Fg vs. distance is a curve, not a straight line. What would a straight-line graph of force vs. distance mean physically? Can you think of any force that works that way?

* If gravity followed a 1/r law instead of a 1/r^2 law -- that is, force decreased with distance instead of distance squared -- how would the graph look different? Would gravity be stronger or weaker at large distances compared to what Newton found?

Big G and the Cavendish Experiment

Newton's equation contained G, but Newton had no way to measure it. He could write $F = Gm_1m_2/r^2$, but without knowing G, he could not calculate actual forces from the equation -- only ratios and proportions. G was the missing number that would turn Newton's law from a beautiful pattern into a precise calculating tool.

For over a century after Newton's death, G remained unmeasured. The problem was devilishly simple to state: to measure G, you need to measure the gravitational force between two objects whose masses and separation you know. But the gravitational force between any two human-scale objects is so absurdly small that it was completely undetectable with the instruments of the time.

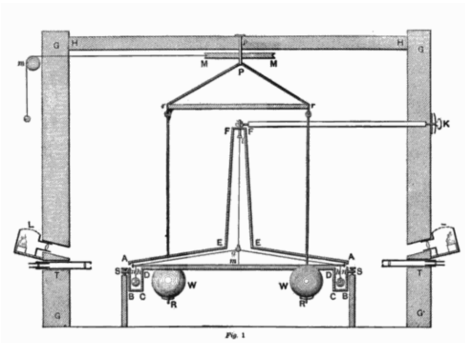


The solution came in 1798, from a brilliant and famously eccentric English scientist named Henry Cavendish. He was 67 years old, a recluse who communicated with his own servants by written notes to avoid having to speak to them. He also happened to be one of the finest experimental physicists who ever lived.

The Torsion Balance

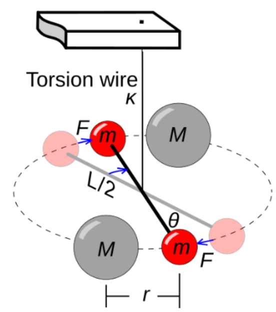
Cavendish's key insight was to eliminate friction entirely. Every previous attempt to measure the tiny gravitational force between small objects had been defeated by friction -- the force was so small that it was completely swamped by the friction in any mechanical bearing or pivot.

His solution was the torsion balance. He attached two small lead balls -- each about 0.73 kg -- to the ends of a wooden rod about 1.8 meters long. Then he hung the rod from its center point using an extremely fine silver wire. The rod was free to rotate with almost zero friction, because it was suspended from the wire rather than resting on any surface. The only resistance to rotation was the slight stiffness of the wire itself -- its torsion, or twisting force.



Cavendish calibrated this torsion -- he measured how much force it took to twist the wire by a given angle. Then he brought two large lead balls -- each about 158 kg -- close to the small ones. The tiny gravitational attraction between the small and large balls caused the rod to twist by a fraction of a degree. By measuring that angle and knowing the torsion of the wire, he could calculate the gravitational force. And from the force, knowing all the masses and distances, he could calculate G.

(Images taken from Wikipedia)

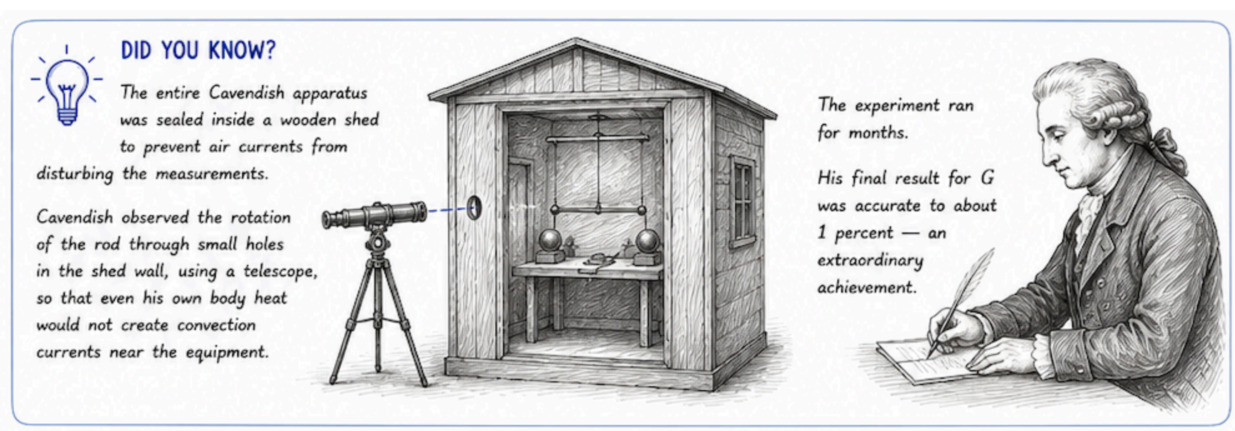


Watch & Explore

> The Cavendish Experiment -- Obvious Gravitational Attraction
> Cavendish Experiment -- Hamilton College Physics
> Cavendish Gravity Experiment Time Lapse -- Version 1
> How Do You Measure the Strength of Gravity? -- NIST

Did You Know?

The entire Cavendish apparatus was sealed inside a wooden shed to prevent air currents from disturbing the measurements. Cavendish observed the rotation of the rod through small holes in the shed wall, using a telescope, so that even his own body heat would not create convection currents near the equipment. The experiment ran for months. His final result for G was accurate to about 1 percent -- an extraordinary achievement.



Cavendish Measured the Mass of the Earth

The most dramatic application of Cavendish's measurement was one he pointed out himself. With G now known, and with the values of g (the gravitational field strength at Earth's surface, measurable by timing falling objects) and R (Earth's radius, known since ancient Greece), you can rearrange the field equation to find Earth's mass:

KEY EQUATION -- Earth's Mass from G

M_Earth = g * R^2 / G

g = 9.8 m/s^2 | R = 6.37 x 10^6 m | G = 6.674 x 10^-11 | Result: M_Earth = 5.97 x 10^24 kg

Nearly six thousand billion billion kilograms. Cavendish called his paper 'Experiments to Determine the Density of the Earth' -- but what he had really done was weigh the world. For the first time in history, a human being had calculated the mass of the planet they were standing on.

Mass Vs. Weight

Weight is the gravitational force acting on an object. Once you know the local value of g, calculating weight is the simplest equation in this chapter:

KEY EQUATION -- Weight

W = mg

W = weight in Newtons | m = mass in kilograms | g = local gravitational field strength (m/s^2)

This means weight is not a fixed property of an object -- it depends on where you are. Your mass is always the same. Your weight changes depending on which planet you are standing on.

Location	Surface g (m/s^2)	Your mass (kg)	Your weight (N)
Earth (surface)	9.8	60 kg	588 N
The Moon	1.6	60 kg	96 N
Mars	3.7	60 kg	222 N
Jupiter (surface)	24.8	60 kg	1488 N
Deep space (no planet)	approx 0	60 kg	approx 0 N

Common Misconception

People often say: *"Mass and weight are the same thing. If I weigh 60 kg, my mass is 60 kg."*

The truth is: Mass is the amount of matter in an object -- measured in kilograms. It never changes regardless of where you are in the universe. Weight is the gravitational force on that mass -- measured in Newtons. It changes completely depending on the local gravitational field. When people say they 'weigh 60 kg' they mean their mass is 60 kg. Their actual weight in Newtons is 60 x 9.8 = 588 N on Earth -- and only 96 N on the Moon.

Watch & Explore

> The difference between mass and the weight - Veritasium

Why the Moon Doesn't Fall towards the Earth? Or does it?

We are now ready to close the loop that Newton opened in his mind on that farm in Lincolnshire. We have the equation. We have the constant. We have the field concept. Now let us use them to answer the question we started with: why doesn't the Moon fall?

The answer, it turns out, is that the Moon does fall. Constantly. Every second of every day, for four and a half billion years. It falls toward Earth at precisely the rate that Earth's surface curves away beneath it.

Did You Know?

Newton calculated that the Moon falls toward the Earth by 1.3 mm every second. That is super tiny—about the thickness of a single penny!

Can you come up with a way of calculating this? What do you need to know in order to calculate this distance?

Watch & Explore

You have probably already watched the Mechanical Universe's Apple and the Moon episode, but if not, watch minutes 20-23 for a wonderful explanation.

> How far does the Moon fall into orbit every second?

Watch & Explore

You have come a very long way! Do you think, given all the information, you can calculate the force between the Moon and the Earth?

> How much is the force of gravity between the Moon and the Earth?

Newton's Cannon: A Thought Experiment

Newton himself explained this with a thought experiment that remains one of the most vivid in all of science. Imagine standing on top of an impossibly tall mountain -- so tall that there is no atmosphere to slow anything down. You fire a cannonball horizontally.

If you fire it gently, it travels a short distance and then falls to Earth in a simple curved arc -- a parabola. Fire it faster, and the parabola becomes longer and flatter. Faster still, and it curves around the Earth, landing far away.

Now keep increasing the speed. At some critical speed, something remarkable happens: the ball travels so fast that by the time gravity has curved its path downward by a meter, the Earth's surface has also curved away beneath it by a meter. The ball never gets closer to the ground. It is in orbit.

Watch & Explore

You have probably already watched the Mechanical Universe's Apple and the Moon episode, but if not, watch minutes 17-20 for a wonderful explanation.

> How much is the force of gravity between the Moon and the Earth?

Think About It

Astronauts on the International Space Station feel completely weightless -- they float freely inside the station, just like the objects around them. But the ISS is only about 400 km above Earth's surface. At that altitude, the gravitational field strength g is still about 8.8 m/s^2 -- nearly as strong as on the ground. If gravity is almost the same up there as it is here, why do astronauts feel weightless?

This is one of the most important questions in all of physics, and the answer is this: the astronauts are not weightless because gravity is weak -- it is almost as strong there as at the surface. They are weightless because they are in free fall. They, the station, and every object inside it are all falling toward Earth at exactly the same rate. Nothing presses against anything else. There is no contact force, and therefore no sensation of weight.

Weight -- the feeling of heaviness -- comes not from gravity itself, but from the surface beneath you pushing back against you. In free fall, nothing pushes back. You feel nothing. You float.

Did You Know?

The ISS travels at about 7.7 km per second -- roughly 28,000 km per hour -- to maintain its orbit. At that speed, it completes one orbit around the Earth every 90 minutes. The crew sees 16 sunrises and 16 sunsets every single day.

Time for a game!

Check your understanding of this chapter through a fun Kahoot game! Make sure the logic is solid before moving on.

> Blooket / Kahoot -- Orbits and Free Fall

Chapter Summary

* Newton's central insight: the force pulling an apple downward and the force holding the Moon in orbit are the same force -- gravity -- acting at different distances.
* The gravitational field is the invisible region of influence surrounding every massive object. It points toward the center of the mass and weakens with distance according to the inverse-square law.
* The inverse-square law: doubling the distance reduces the force by a factor of 4. Tripling it reduces the force by a factor of 9. Gravity has infinite range but grows arbitrarily weak at large distances.
* Newton's Law of Universal Gravitation: $F = Gm1m2/r^2$. The force depends on both masses equally (Newton's Third Law symmetry) and on the inverse square of the center-to-center distance.
* Big G = 6.674 x 10^-11 N*m^2/kg^2 is the Universal Gravitational Constant -- the same everywhere in the universe. It is tiny, which is why gravity between human-scale objects is undetectable.
* The Cavendish experiment (1798) measured G for the first time using a torsion balance to detect the tiny gravitational attraction between lead balls. The payoff: calculating Earth's mass as 5.97 x 10^24 kg.
* Little g = GM/R^2 is the gravitational field strength at a planet's surface. It is different for every planet. At Earth's surface, $g = 9.8\ m/s^2$.
* Weight = mg is the gravitational force on an object. Mass is fixed everywhere; weight changes with the local value of g.

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, whilst the great ocean of truth lay all undiscovered before me." -- Isaac Newton

Check Your Understanding

Conceptual Questions

1. Newton's insight was that the apple and the Moon are governed by the same force. What evidence, available to Newton in 1666, supported the idea that gravity reaches all the way to the Moon?
2. Two objects are separated by a distance r and exert a gravitational force F on each other. What happens to the force if: (a) the distance doubles; (b) one mass triples; (c) both masses double and the distance halves?
3. The gravitational field strength at Earth's surface is g = 9.8 m/s^2. At a height equal to Earth's radius above the surface (so the distance from Earth's center is 2R), what is the value of g? Explain using the inverse-square law.
4. An astronaut is floating weightlessly inside the ISS. A friend on the ground says 'there is no gravity up there.' Write a careful correction of this statement -- explain what is actually happening.
5. Gravity follows an inverse-square law. So does the intensity of light from a lamp. What do these two phenomena have in common that explains why both follow the same mathematical pattern?
6.

Calculation Problems

7. Calculate the gravitational force between Earth (M = 5.97 x 10^24 kg) and the Moon (m = 7.34 x 10^22 kg), given that the average Earth-Moon distance is 3.84 x 10^8 m. Use G = 6.674 x 10^-11.
8. A 70 kg person stands on the surface of Mars. The surface gravitational field strength on Mars is 3.7 m/s^2. (a) What is their weight on Mars? (b) What is their weight on Earth? (c) Has their mass changed?
9. The International Space Station orbits at an altitude of 400 km. Earth's radius is 6,370 km. Use v = sqrt(GM/r) to calculate the orbital speed of the ISS. How does this compare to the speed of a bullet (about 400 m/s)?
10. Using the Cavendish result G = 6.674 x 10^-11, the surface gravity g = 9.8 m/s^2, and Earth's radius R = 6.37 x 10^6 m, calculate Earth's mass using M = gR^2/G. Express your answer in scientific notation.

A Question to Carry Into Chapter 3

We showed in this chapter that all objects fall at the same rate near Earth's surface -- a feather and a hammer, a tennis ball and a bowling ball, all accelerate at the same g. We explained it by saying: the increased gravitational pull on a heavier object is exactly canceled by its increased inertia.

But here is a deeper question: why should these two completely different properties of matter -- gravitational mass (how strongly an object is pulled by gravity) and inertial mass (how strongly an object resists being accelerated) -- be exactly equal? There is no obvious reason they should be. Newton simply noticed that they are. Einstein, two and a half centuries later, realized this was not a coincidence. It was trying to tell us something profound about the nature of gravity itself.

That is where Chapter 3 begins.

Check Your Understanding - Answer Key

Conceptual Questions

1. Newton's insight was that the apple and the Moon are governed by the same force. What evidence, available to Newton in 1666, supported the idea that gravity reaches all the way to the Moon?

Newton had three pieces of evidence pointing toward this conclusion. Newton showed mathematically that this pattern is only consistent with a force that weakens as $1/r^2$. If that force governed the planets, it must reach across enormous distances of empty space. Second, the Moon was visibly accelerating toward Earth — it was not traveling in a straight line as it would if no force acted on it, but curving continuously around Earth. Something was pulling it inward. Third, Newton calculated the rate at which the Moon was falling toward Earth (about 0.00272 m/s^2) and compared it to the surface gravity of 9.8 m/s^2. The Moon is about 60 Earth radii away, so the inverse-square law predicts gravity there should be $9.8 / 60^2 \approx 0.00272$ m/s^2 — which matched the Moon's observed inward acceleration almost exactly. This agreement was the key evidence: the same force, weakened by distance, explained both.

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2. Two objects are separated by a distance r and exert a gravitational force F on each other. What happens to the force if: (a) the distance doubles; (b) one mass triples; (c) both masses double and the distance halves?

Starting from $F = Gm_1m_2/r^2$:

(a) Distance doubles ($r \to 2r$): New force = $Gm_1m_2/(2r)^2 = Gm_1m_2/4r^2 = F/4$. The force drops to one quarter of its original value.

(b) One mass triples ($m_1 \to 3m_1$): New force = $G(3m_1)m_2/r^2 = 3Gm_1m_2/r^2 = 3F$. The force triples. Gravitational force is directly proportional to each mass.

(c) Both masses double AND distance halves ($m_1 \to 2m_1$, $m_2 \to 2m_2$, $r \to r/2$): New force = $G(2m_1)(2m_2)/(r/2)^2 = 4Gm_1m_2/(r^2/4) = 16Gm_1m_2/r^2 = 16F$. The force becomes 16 times stronger. The two doubled masses contribute a factor of 4, and halving the distance contributes another factor of 4 (inverse-square law), giving $4 \times 4 = 16$.

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3. The gravitational field strength at Earth's surface is $g = 9.8$ m/s^2. At a height equal to Earth's radius above the surface (so the distance from Earth's center is 2R), what is the value of g?

At Earth's surface, the distance from Earth's center is R. At the given height, the distance from Earth's center is $R + R = 2R$ — the distance has doubled.

By the inverse-square law, doubling the distance reduces g by a factor of $2^2 = 4$.

g at 2R = 9.8 / 4 = **2.45 m/s^2**

This is still substantial — about one quarter of surface gravity. The ISS orbits at roughly 400 km altitude, which is much less than one Earth radius above the surface, so gravity there is about 8.8 m/s^2 — still nearly 90% of what it is on the ground. This is why the "no gravity in space" claim is a misconception.

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4. An astronaut is floating weightlessly inside the ISS. A friend on the ground says "there is no gravity up there." Write a careful correction of this statement.

The friend's statement is incorrect in two ways. First, gravity at ISS altitude (about 400 km above the surface) is still approximately 8.8 m/s^2 — nearly 90% as strong as on the ground. The gravitational field does not switch off in space; it weakens gradually following the inverse-square law, and 400 km is a tiny distance compared to Earth's radius of 6,370 km.

Second, the reason the astronaut feels weightless is not the absence of gravity but the presence of free fall. The ISS and everything inside it — the astronaut, the food, the equipment — are all falling toward Earth at exactly the same rate. Nothing presses against anything else, so there is no contact force, and therefore no sensation of weight. Weight — the feeling of heaviness — comes not from gravity itself but from the surface beneath you pushing back. In free fall, nothing pushes back. The astronaut is not experiencing zero gravity. They are experiencing zero contact force — which feels identical to zero gravity, but is a completely different physical situation.

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5. Gravity follows an inverse-square law. So does the intensity of light from a lamp. What do these two phenomena have in common that explains why both follow the same mathematical pattern?

Both gravity and light spread outward in all directions from a point source through three-dimensional space. As they spread, the same total amount of influence — whether gravitational field or light energy — must cover a larger and larger surface area as the distance increases. The surface area of a sphere grows as $4\pi r^2$, so the amount of influence per unit area decreases as $1/r^2$. This is purely a consequence of the geometry of three-dimensional space, not a specific property of either gravity or light. Any quantity that spreads equally in all directions from a point source through three-dimensional space will follow an inverse-square law. This is why radio signals, sound from a point source, and electric force between charges all follow the same $1/r^2$ pattern.

Calculation Problems — Answer Key

1. Calculate the gravitational force between Earth and the Moon.

Given:

- $G = 6.674 \times 10^{-11}\ N \cdot m^2/kg^2$
- $M = 5.97 \times 10^{24}$ kg
- $m = 7.34 \times 10^{22}$ kg
- $r = 3.84 \times 10^8$ m

$F = Gm_1m_2/r^2$

$F = (6.674 \times 10^{-11}) \times (5.97 \times 10^{24}) \times (7.34 \times 10^{22}) / (3.84 \times 10^{8})^2$

Step 1 — numerator: $6.674 \times 5.97 \times 7.34 = 292.2$ (combining the numbers) $10^{-11} \times 10^{24} \times 10^{22} = 10^{35}$ Numerator $= 292.2 \times 10^{35} = 2.922 \times 10^{37}$

Step 2 — denominator: $(3.84 \times 10^{8})^2 = 14.75 \times 10^{16} = 1.475 \times 10^{17}$

Step 3 — divide: $F = 2.922 \times 10^{37} / 1.475 \times 10^{17} =$ **1.98×10^{20} N**

This is approximately 2×10^{20} Newtons — an almost unimaginably large force, which is why the Moon stays locked in its orbit despite traveling at over 1 km per second.

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2. A 70 kg person on Mars. Surface gravity = 3.7 m/s².

(a) Weight on Mars: $W = mg = 70 \times 3.7 =$ **259 N**

(b) Weight on Earth: $W = mg = 70 \times 9.8 =$ **686 N**

(c) Has their mass changed? No. Mass is the amount of matter in an object and does not change with location. The person's mass is 70 kg on Mars, on Earth, on the Moon, or in deep space. Only their weight — the gravitational force acting on them — changes, because weight depends on the local gravitational field strength g, which varies from planet to planet.

On Mars they feel about 38% of their Earth weight — noticeably lighter, but far from weightless.

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3. Orbital speed of the ISS.

Given:

- $G = 6.674 \times 10^{-11}$
- $M = 5.97 \times 10^{24}$ kg
- Altitude = 400 km = 4.00×10^{5} m
- Earth's radius = 6,370 km = 6.37×10^{6} m
- $r = R + \text{altitude} = 6.37 \times 10^{6} + 4.00 \times 10^{5} = 6.77 \times 10^{6}$ m

$v = \sqrt{(GM/r)}$

$GM = 6.674 \times 10^{-11} \times 5.97 \times 10^{24} = 3.984 \times 10^{14}$

$GM/r = 3.984 \times 10^{14} / 6.77 \times 10^{6} = 5.885 \times 10^{7}$

$v = \sqrt{(5.885 \times 10^{7})} =$ **7,671 m/s ≈ 7.67 km/s**

Comparison to a bullet: A typical bullet travels at about 400 m/s. The ISS travels at 7,671 m/s — roughly **19 times faster than a bullet**. It completes one full orbit around Earth approximately every 90 minutes, meaning the crew sees 16 sunrises and 16 sunsets every single day.

4. Calculate Earth's mass using $M = gR^2/G$.

Given:

- $g = 9.8\ m/s^2$
- $R = 6.37 \times 10^6\ m$
- $G = 6.674 \times 10^{-11}$

$M = gR^2/G$

Step 1 — R^2: $(6.37 \times 10^6)^2 = 40.58 \times 10^{12} = 4.058 \times 10^{13}\ m^2$

Step 2 — gR^2: $9.8 \times 4.058 \times 10^{13} = 39.77 \times 10^{13} = 3.977 \times 10^{14}$

Step 3 — divide by G: $M = 3.977 \times 10^{14} / 6.674 \times 10^{-11}$

$= (3.977 / 6.674) \times 10^{14+11}$

$= 0.5959 \times 10^{25}$

= **5.96×10^{24} kg**

This matches the accepted value of 5.97×10^{24} kg almost exactly. The tiny difference comes from rounding in our values of g and R. What Cavendish actually did in 1798 was measure G with his torsion balance, then plug it into exactly this calculation to produce the first reliable estimate of Earth's mass — earning his experiment the nickname "weighing the world."